ECE 343: Signals and Systems

Test 1: Continuous-Time Signals and Systems

1. Let $x(t) = Ae^{(\sigma+j\omega)t}$, where $A > 0$, σ, ω, and t are all real.
   (a) Find the real and imaginary parts of $x(t)$.
   (b) Find the modulus (magnitude) and the argument (angle) of $x(t)$.
   (c) Find the hermitian and antihermitian (skew hermitian) parts of $x(t)$. If they are complex, put them in rectangular form.

2. State whether or not the following systems are linear and/or time invariant. For all parts, $y(t)$ is the output of the system and $x(t)$ is the input. Assume x and y are complex-valued unless otherwise indicated. For full credit, give some justification for your answer.
   (a) $y(t) = x^*(t)$
   (b) $y(t) = x(t) + \int_{-1}^{1} x_o(\lambda)d\lambda$, where $x_o(t)$ is the odd part of $x(t)$.
   (c) $y(t) = x^2(t) = x(t) \cdot x(t)$, with $x, y \in \mathrm{GF}(2)$. The elements of GF(2) are not complex or real numbers, rather they act like binary values in a digital system. In GF(2), there are only two possible values, 0 or 1, and addition and multiplication are logical XOR and logical AND, respectively.
   $$0 + 0 = 1 + 1 = 0$$
   $$1 + 0 = 0 + 1 = 1$$
   $$0 \cdot 0 = 1 \cdot 0 = 0 \cdot 1 = 0$$
   $$1 \cdot 1 = 1$$
   (d) $y(t) = x(t) - x(-t)$

3. Let $h_0(t) = u(t)$ and $h_n(t) = h_{n-1}(t) * u(t)$, where $u(t)$ is the unit step function.
   (a) Find and sketch $h_1(t) = u(t) * u(t)$.
   (b) Find and sketch $h_2(t) = h_1(t) * u(t)$.
   (c) (Bonus) Find $h_n(t)$ for any integer $n \geq 0$.

4. Let a system with input $x(t)$ and output $y(t)$ be defined by
   $$y(t) = \int_{t-1}^{t} (1+\lambda)\, x(\lambda)d\lambda - t\int_{t-1}^{t} x(\lambda)d\lambda.$$
   This system is LTI.
   (a) Find and sketch the impulse response $h(t)$.
   (b) Find and sketch the step response of the system. That is, find the output when the input is $u(t)$.

5. Define an inner product

$$\langle x, y\rangle = \int_{-1}^{1} x(t)y^*(t)\frac{1}{\sqrt{1-t^2}}dt$$

for functions defined on the interval $[-1, 1]$. Are the functions

$$\begin{aligned} T_1(t) &= t \\ T_3(t) &= 4t^3 - 3t \end{aligned}$$

orthogonal with respect to this inner product? You may find the following antiderivatives useful

$$\begin{aligned} \int \frac{1}{\sqrt{1-t^2}}dt &= \sin^{-1}(t) \\ \int \frac{t}{\sqrt{1-t^2}}dt &= -\sqrt{1-t^2} \\ \int \frac{t^2}{\sqrt{1-t^2}}dt &= \frac{1}{2}\sin^{-1}(t) - \frac{1}{2}t\sqrt{1-t^2} \\ \int \frac{t^3}{\sqrt{1-t^2}}dt &= -\frac{1}{3}(t^2+1)\sqrt{1-t^2} \\ \int \frac{t^4}{\sqrt{1-t^2}}dt &= \frac{3}{8}\sin^{-1}(t) - \left(\frac{1}{4}t^3 + \frac{3}{8}t\right)\sqrt{1-t^2} \end{aligned}$$